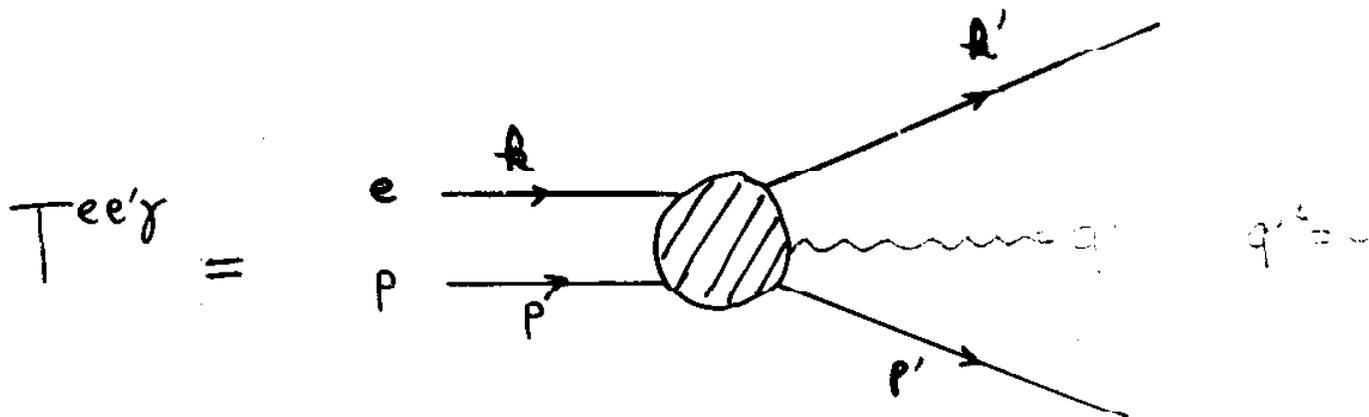


Virtual Compton Scattering



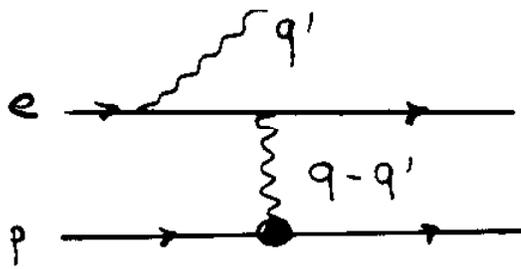
Relevant variables:

$$q = k - k' \quad Q^2 = -q^2 \neq 0$$

$$s = (p + q)^2 = m_N^2 + Q^2 \frac{1 - x_B}{x_B}$$

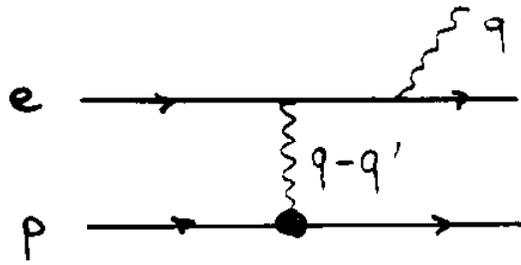
$$t = (p - p')^2$$

$$T_{ee'\gamma} =$$

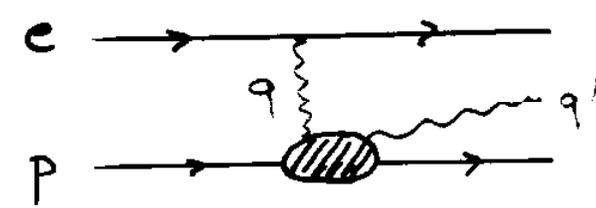


Bethe Heitler process

+

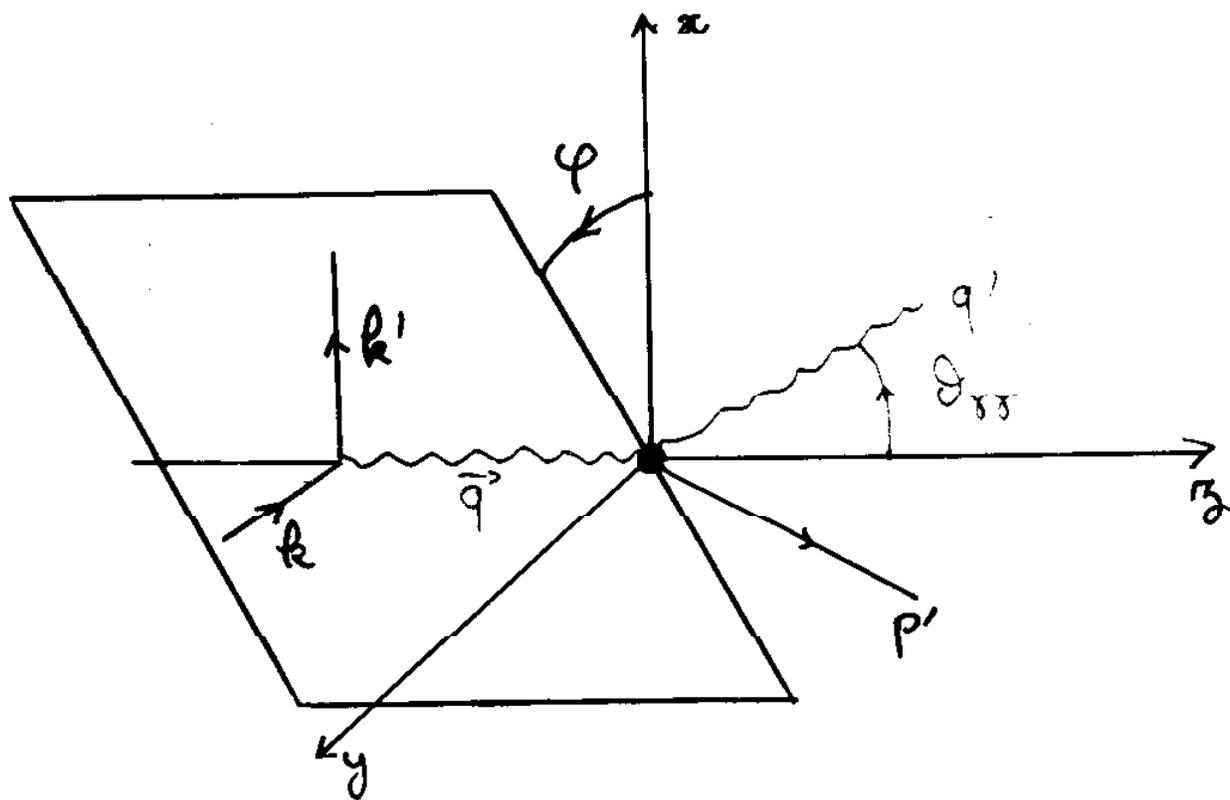


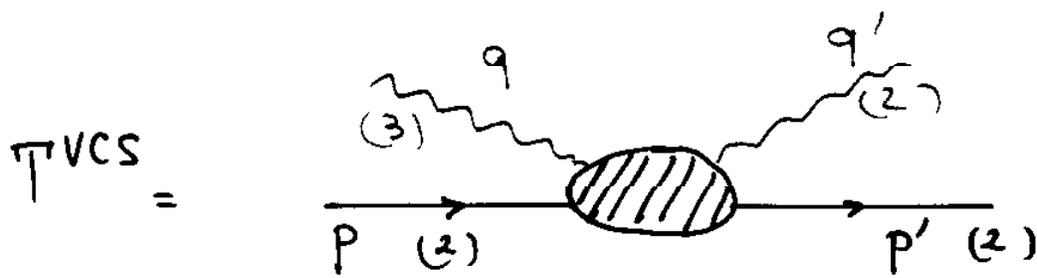
+



VCS

N.B. : The exchanged photon has not the same momentum in the B.H and VCS





$$3 \times 2 \times 2 \times 2 \times \left(\frac{1}{2} \text{ for parity}\right) \\ = 12 \text{ degrees of freedom}$$

12 complex functions of $(q^2, \lambda, \theta_{rr})$

+ interference with BH

→ In general it is hopeless to make a quantitative analysis

→ Look for simplifying situations

① $\sqrt{s} < m_N + m_\pi$ ($\forall Q^2$)

→ Generalized polarizabilities

② (s, t, u) large

→ PQCD (Brodsky ----)

③ (Q^2, s) large, x_B finite, t small

→ Deeply Virtual Compton Scatt.
DVCS

- Generalized parton distributions
- Spin structure
- etc... ?

INCLUSIVE DIS

$$\sigma \sim \sum_X \left| \begin{array}{c} \text{wavy } q \\ \nearrow \\ \text{blob} \\ \searrow \text{ } \\ \text{ } \end{array} \right|^2$$

$$\sim \text{Im} \left(\begin{array}{c} \text{wavy } q \\ \nearrow \\ \text{blob} \\ \searrow \text{ } \\ \text{wavy } q \\ \nearrow \\ \text{blob} \\ \searrow \text{ } \\ P \end{array} \right)$$

$$P = P'$$

EXCLUSIVE DVCS

$$\sigma \sim \left| \begin{array}{c} \text{wavy } q \\ \nearrow \\ \text{blob} \\ \searrow \text{ } \\ \text{wavy } q' \\ \nearrow \\ \text{blob} \\ \searrow \text{ } \\ P' \end{array} \right|^2$$

$$q'^2 = 0$$

$$P' \neq P$$

- Is it possible to go beyond formal resemblance?
- Can we analyse DVCS in the same way as DIS?
(Scaling structure functions, parton interpretation, gluon radiation, etc...)

Relevant literature

1) K. Watanabe Prog. Th. Phys. 67 (1982) 1834

2) X. D. Ji Phys. Rev. Lett. 78 (1997) 610

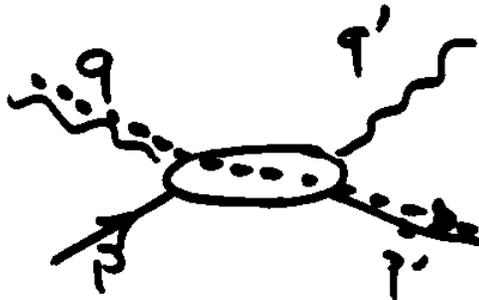
3) A. Radyushkin Phys. Lett. B 380 (1996) 417

$$P = (\vec{0}) \quad q = \begin{pmatrix} Q \\ q \end{pmatrix} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} q \approx \frac{Q^2}{2Mx_B} + Mx_B$$

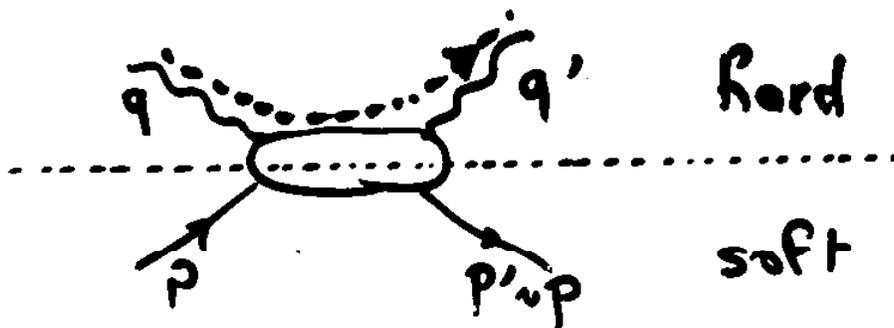
$$q' = \frac{\Delta - M^2}{2(M + q^0 - q \cos \theta)}$$

$$\approx \frac{Q^2(1-x_B)}{2x_B} \frac{1}{M(1-x_B \cos \theta) + \frac{Q^2}{2Mx_B}(1-\cos \theta) + \dots}$$

θ large : $q' \sim M$



θ small ($\ll M/Q$)



$$T^{VCS} = \int d^4u e^{-iq \cdot u} \langle p' | T(J_\mu(0) J_\nu(u)) | p \rangle \epsilon_{(q)}^\mu \epsilon_{(q')}^{\nu*}$$

Lab.: $\vec{q} = \begin{pmatrix} q^0 \\ 0 \\ 0 \\ q \end{pmatrix}$

$$q^0 = \frac{Q^2}{2m\alpha_B}$$

$$q = \sqrt{Q^2 + q_0^2} = \frac{Q^2}{2m\alpha_B} + m\alpha_B + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

$$q \cdot u = \frac{Q^2}{2m\alpha_B} (u^0 - u^3) - m\alpha_B u^3$$

$$= \frac{Q^2}{m\alpha_B} u^- - m\alpha_B (u^+ - u^-) ; \quad u^\pm = \frac{u^0 \pm u^3}{2}$$

① Bjorken limit: $Q^2 \rightarrow \infty$ α_B finite

$$\int d^4u e^{iq \cdot u} \dots$$

dominated by

$$u^- \sim \frac{m\alpha_B}{Q^2} \quad u^+ \sim \frac{1}{m\alpha_B}$$

② $\int d^4u e^{-iq \cdot u} \langle p' | T(J_\mu(0) J_\nu(u)) | p \rangle \equiv$

$$\int d^4u e^{-iq \cdot u} \langle p' | [J_\mu(0), J_\nu(u)] | p \rangle (1 - \theta(u^0))$$

Causality $\Rightarrow [J_\mu(0), J_\nu(u)] = 0$ if $u^2 < 0$

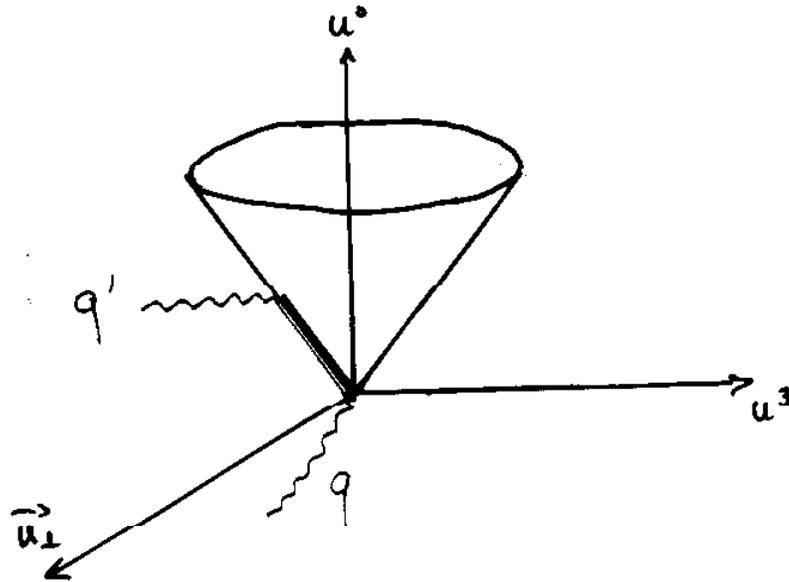
$\Rightarrow \int d^4u e^{-iq \cdot u} (\dots)$ dominated by $0 < u^0^2 - \vec{u}^2 = 4u^-u^+ - \vec{u}_\perp^2$

$$\Rightarrow \vec{u}_\perp^2 < 4u^-u^+ \sim \frac{1}{Q^2}$$

Join $e \cdot \langle p | (j_0(x) j_0(y)) | p \rangle$...

$$U^- \sim \frac{m^2 B}{Q^2}, \quad |U_\perp| \sim \frac{1}{Q}, \quad U^+ \sim \frac{1}{m^2 B}$$

(in B_j limit)



The 4-fold integral is dominated by the red line cone segment $\parallel n$ $n^\mu = \begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix}$

Line segment = zero measure in \mathbb{R}^4

\Rightarrow either $\left\{ \begin{array}{l} \text{TVCS} = 0 \end{array} \right.$

$J(0) J(\omega)$ singular on the light cone.

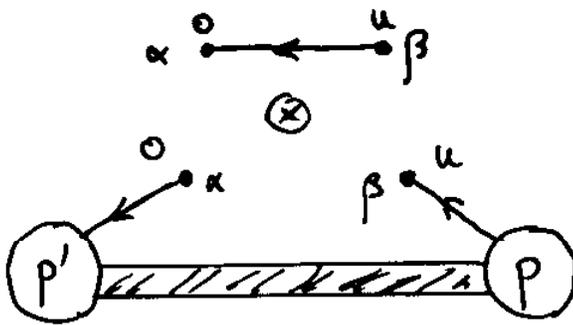
In QCD leading singularity = free field theory

Wick th.

$$\begin{aligned}
 & \langle p' | T (\bar{q}(0) \gamma^\mu q(0) \bar{q}(u) \gamma^\nu q(u)) | p \rangle = \\
 & \overbrace{q_\alpha(0) \bar{q}_\beta(u)} \langle p' | : (\bar{q}(0) \gamma^\mu)_\alpha (\gamma^\nu q(u))_\beta : | p \rangle \\
 & + \overbrace{\bar{q}_\alpha(0) q_\beta(u)} \langle p' | : (\gamma^\mu q(0))_\alpha (\bar{q}(u) \gamma^\nu)_\beta : | p \rangle \\
 & \qquad \qquad \qquad + \text{less sing. terms.}
 \end{aligned}$$

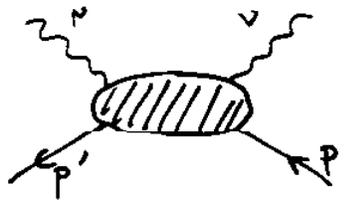
free quark propagator

non perturbative part



$$\int \frac{d^4 k}{(2\pi)^4} \frac{(\gamma \cdot k)_{\alpha\beta} e^{i k \cdot u}}{k^2 + i\epsilon}$$

Diracology, algebra, keep leading terms in $1/Q \rightarrow$



A Feynman diagram showing a fermion loop (represented by a shaded oval) with two external fermion lines (momenta p' and p) and two external photon lines (momenta μ and ν).

$$= T_{\mu}^{\rho\nu} + T_{\nu}^{\rho\mu}$$

$$T_{\Delta}^{\mu\nu} = \frac{1}{2} (\bar{n}^{\rho} n^{\nu} + \bar{n}^{\nu} n^{\rho} - g^{\rho\nu}) \int dx \left(\frac{1}{x - \chi + i\epsilon} + \frac{1}{x + \chi + i\epsilon} \right) \\ \times \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{q}(-\frac{\lambda n}{2}) \gamma \cdot n q(\frac{\lambda n}{2}) | p \rangle$$

$$T_{\Delta}^{\mu\nu} = -\frac{i}{2} \epsilon^{\rho\nu\alpha\beta} \bar{n}_{\alpha} n_{\beta} \int dx \left(\frac{1}{x - \chi + i\epsilon} - \frac{1}{x + \chi + i\epsilon} \right) \\ \times \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{q}(-\frac{\lambda n}{2}) \gamma \cdot n \gamma^5 q(\frac{\lambda n}{2}) | p \rangle$$

$$n^{\rho} \sim \begin{pmatrix} 1 \\ 0_{\perp} \\ -1 \end{pmatrix} ; \quad \bar{n}^{\rho} \sim \begin{pmatrix} 1 \\ 0_{\perp} \\ 1 \end{pmatrix} ; \quad \chi = \frac{Q^2}{2(p+p') \cdot q}$$

OFF Forward Parton Distributions (OFFPD)

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{q}(-\frac{\lambda n}{2}) \gamma^\mu q(\frac{\lambda n}{2}) | p \rangle =$$

$$\bar{U}(p') \left[H(x, x, t) \gamma^\mu + E(x, x, t) \frac{i\sigma^{\mu\nu} \Delta_\nu}{2m} \right] U(p)$$

$$(\Delta = p' - p)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{q}(-\frac{\lambda n}{2}) \gamma^\mu \gamma^5 q(\frac{\lambda n}{2}) | p \rangle =$$

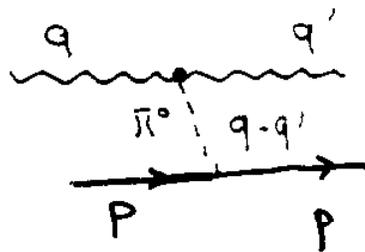
$$\bar{U}(p') \left[\tilde{H}(x, x, t) \gamma^\mu \gamma^5 + \tilde{E}(x, x, t) \frac{\Delta^\nu \gamma^5}{2m} \right] U(p)$$

WARNING!

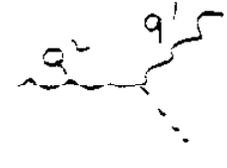
In DIS $Q^2 \sim 2 \text{ GeV}^2$ is enough for scaling.

but DIS sees only $\text{Im}(T^{\mu\nu})$

Real contributions like



do not contribute to DIS

That is higher twist because  $\sim \frac{1}{Q^2}$

but t small $\rightarrow \frac{1}{t - m_{\pi^2}}$ large

REAL PART of DVCS may not scale
as fast as IMAGINARY PART

① 013

$$p' = p \rightarrow \begin{aligned} H(x, 0, 0) &= q(x) \\ \tilde{H}(x, 0, 0) &= \Delta q(x) \end{aligned} \quad \text{(quark distributions)}$$

useless because $p' = p \Rightarrow Q^2 = 0$ kinematically

② Elastic form factors

$$\int dx H(x, x, t) = F_1(t) \quad \text{(per flavor)}$$

$$\int dx E(x, x, t) = F_2(t)$$

$$\int dx \tilde{H}(x, x, t) = g_A(t)$$

$$\int dx \tilde{E}(x, x, t) = h_A(t)$$

useful to normalize model calculations of OFFD's

③ Spin structure

$$\int dx x [H(x, x, t) + E(x, x, t)] = A(t) + B(t)$$

$$\frac{1}{2} [A(0) + B(0)] = J_q = \text{Total spin} \\ \text{(intrinsic + orbital)} \\ \text{carried by the quarks.}$$

Assume $H(x, x, t) = q(x) h(t)$
 $E(x, x, t) = q(x) e(t)$
 etc...

at least correct when $p' \rightarrow p$

Determine $h(t), e(t), \dots$ through the sum rules

$$\int dx H(x, x, t) = F_1(t)$$

$$\begin{cases} q_v(x) = \frac{N}{\sqrt{x}} (1-x)^3 & \int dx q_v(x) = 1 \text{ by convention} \\ q_s(x) = \frac{N}{8x} (1-x)^7 & \text{(mostly irrelevant here)} \end{cases}$$

Take flavour into account:

$$h(t) = F_1^p(t) + \frac{2}{3} F_1^n(t)$$

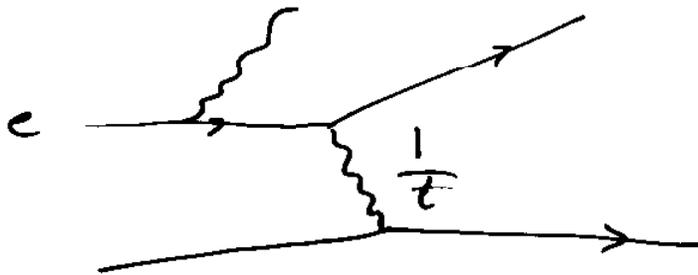
$$e(t) = F_2^p(t) + \frac{2}{3} F_2^n(t)$$

$$\tilde{h}(t) = \frac{1}{3} g_A(t)$$

$$\tilde{e}(t) = \frac{1}{6} h_A(t) \quad (\text{assuming } h_A^{\text{box}} = 0)$$

Summary for cross sections

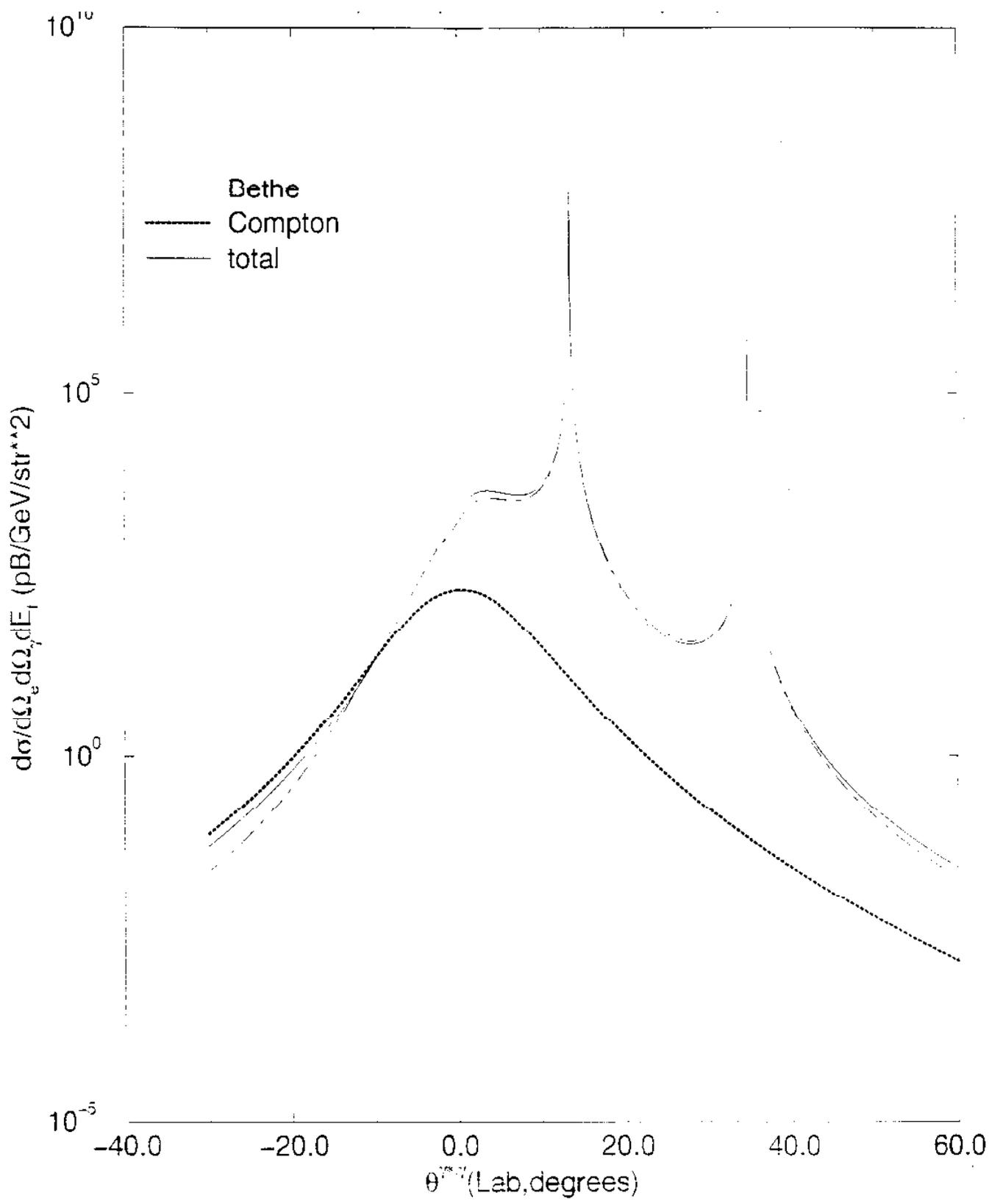
- 1) Need small t/q^2 for a chance of scaling
- 2) DVCS/BH $\sim t/q^2$



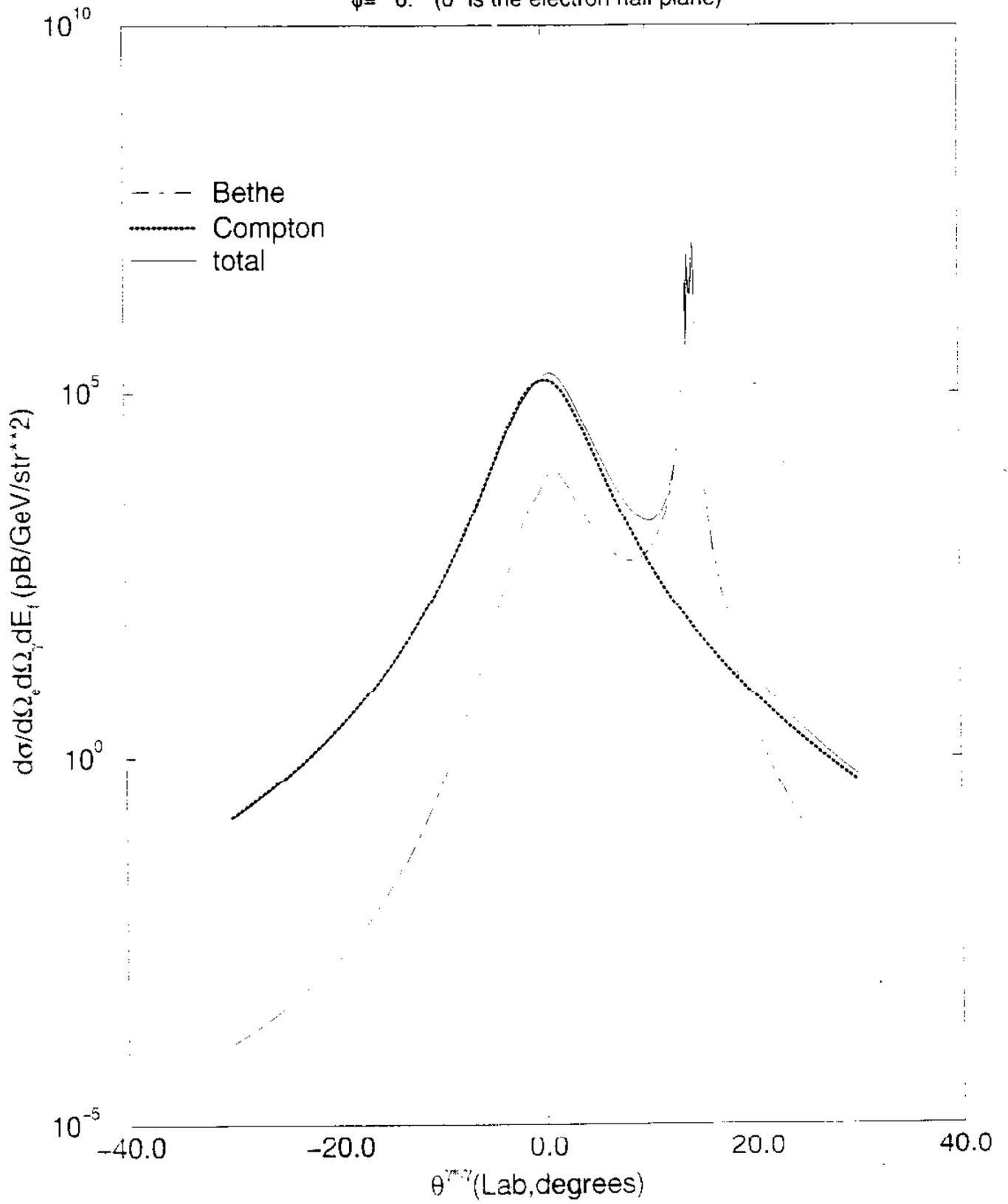
only way to kill BH: increase beam energy

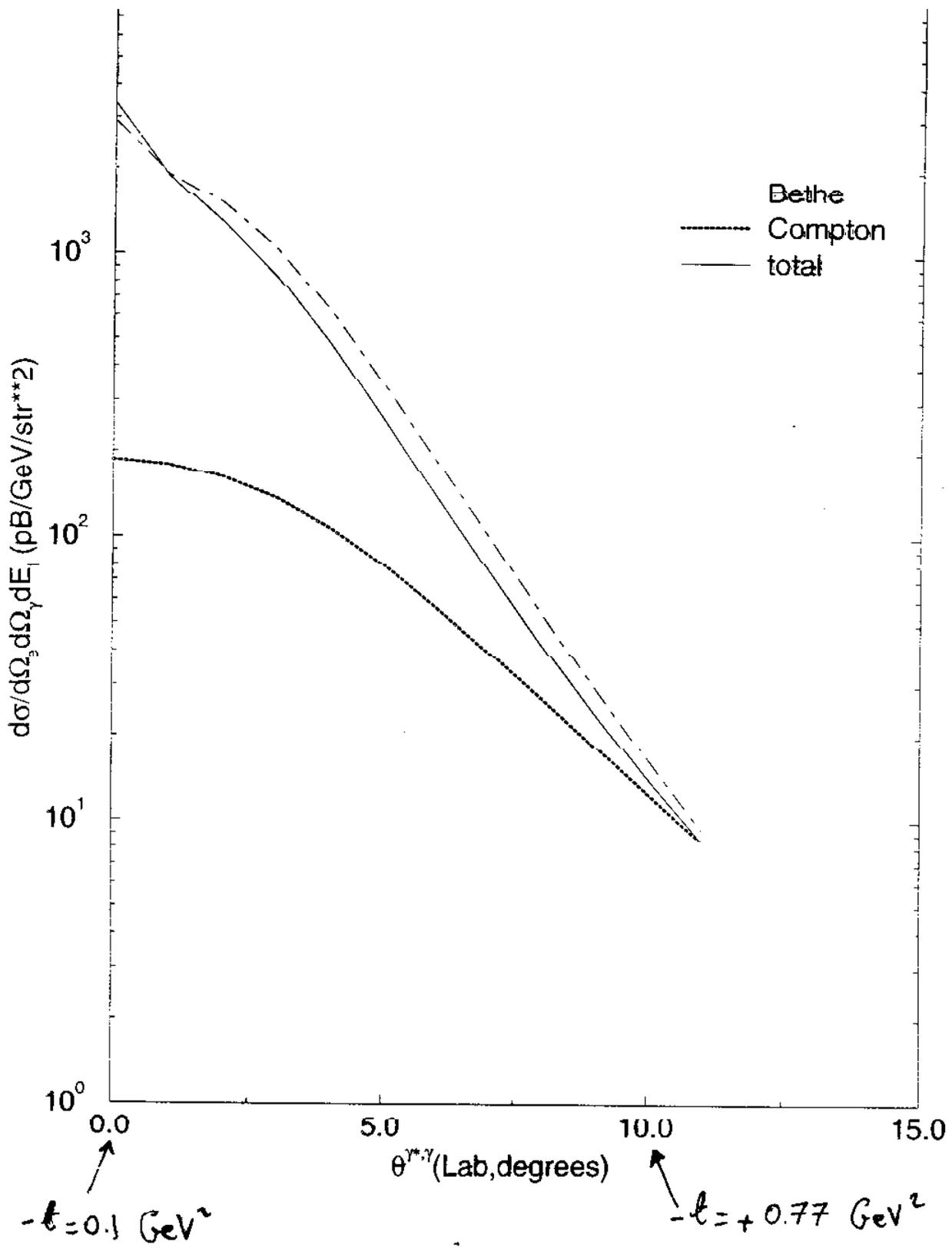
Cebaf (6 GeV) : too short

Compass (100-200 GeV) . looks good.

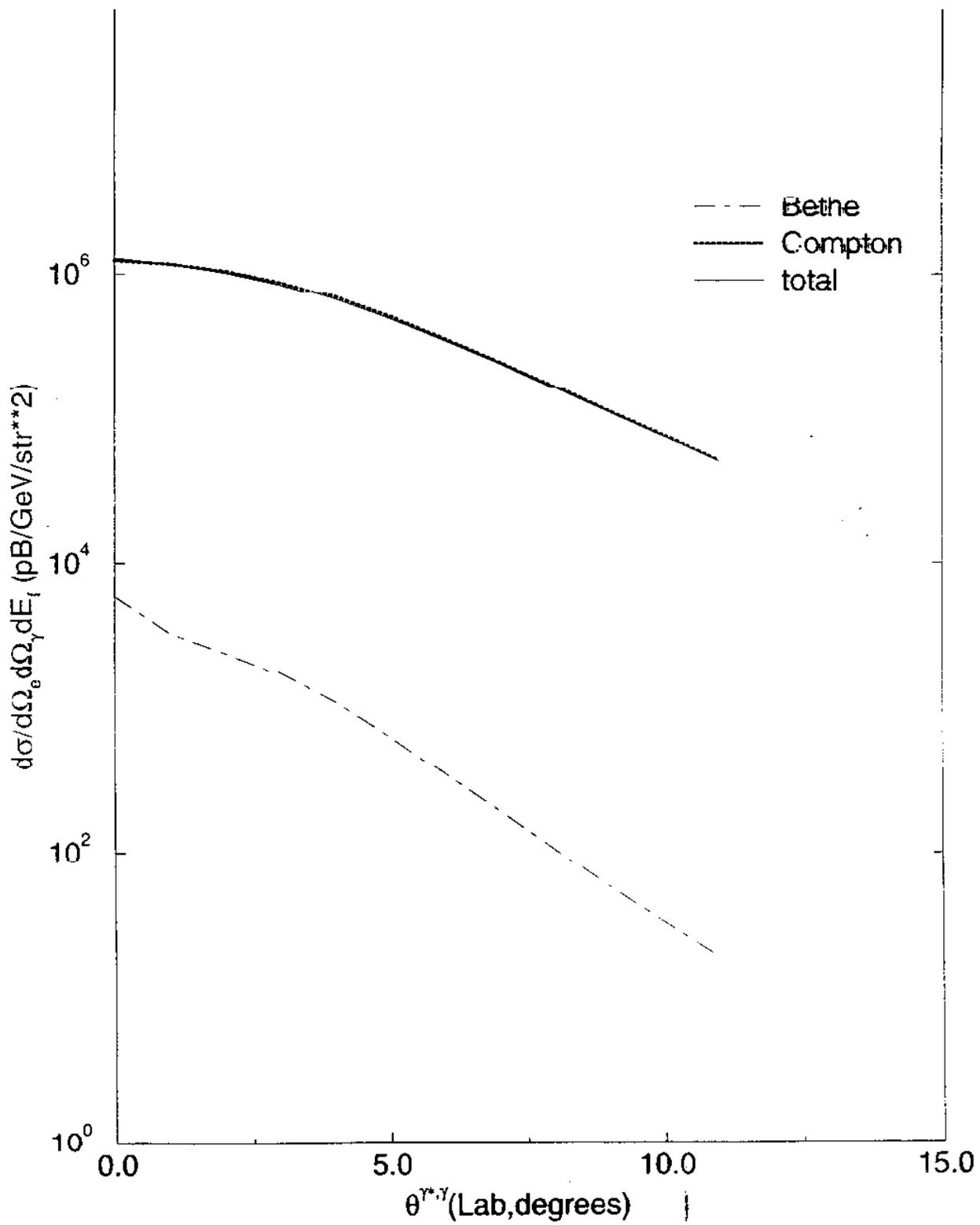


$\phi = 0^\circ$ (0° is the electron half plane)





$$0.145 < x < 0.15$$



$-t = 0.07 \text{ GeV}^2$

$-t = 0.77$

$0.144 < \alpha < 0.16$

Consider single spin asymmetry $\Delta\sigma = \sigma(\uparrow) - \sigma(\downarrow)$

$$\Delta\sigma = \left| \begin{array}{c} \text{e} \rightarrow \\ \text{p} \rightarrow \end{array} \right. \left. \begin{array}{c} \nearrow \\ \rightarrow \end{array} \right|^2 - \left| \begin{array}{c} \text{e} \leftarrow \\ \text{p} \rightarrow \end{array} \right. \left. \begin{array}{c} \nearrow \\ \rightarrow \end{array} \right|^2$$

$$\sigma^{\text{BH+DVCS}} = \sigma^{\text{BH}} + \sigma^{\text{DVCS}} + \sigma^{\text{Interf.}}$$

• BH real $\Rightarrow \sigma^{\text{BH}}(\uparrow) = \sigma^{\text{BH}}(\downarrow)$

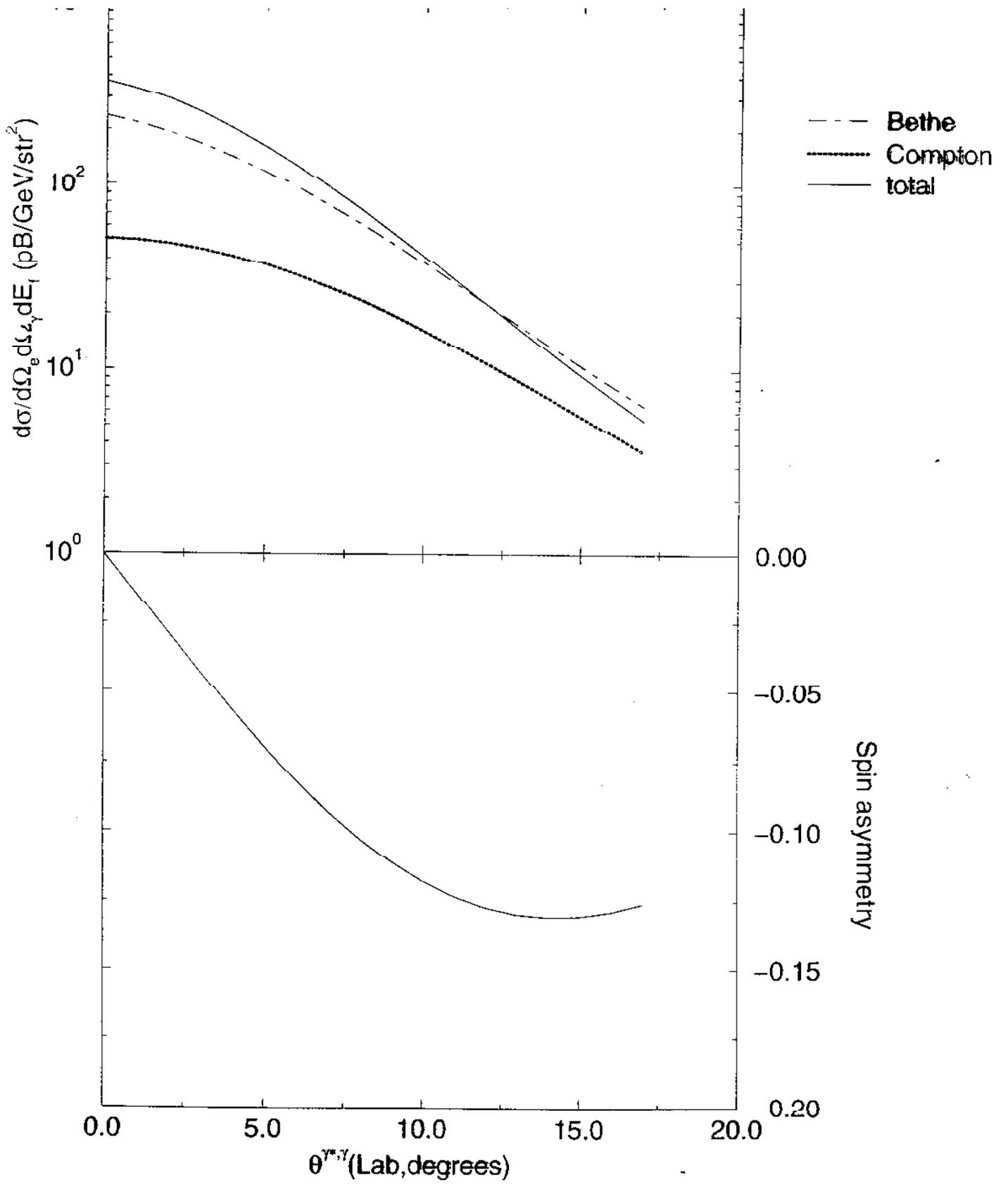
• Leading twist Lorentz structure of DVCS \Rightarrow

$$\sigma^{\text{DVCS}}(\uparrow) - \sigma^{\text{DVCS}}(\downarrow)$$

$$\boxed{\Delta\sigma^{\text{BH+DVCS}} = \Delta\sigma^{\text{Interf}}}$$

$$\Delta\sigma^{\text{Interf.}} \sim \sigma^{\text{BH}} \sigma^{\text{DVCS}*} + \text{cc}$$

$$\Rightarrow \boxed{\begin{array}{l} \text{even if } \sigma^{\text{BH}} \gg \sigma^{\text{DVCS}} \text{ (Ceaf case)} \\ \Delta\sigma \text{ is linear in } \sigma^{\text{DVCS}}, \text{ hence in } H, E, \tilde{H}, \tilde{E} \end{array}}$$

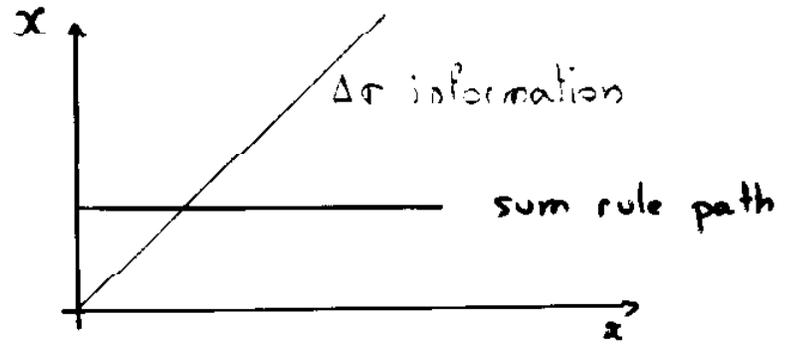


$$\mathbb{T}^{\text{DVCS}} \sim \int dx \frac{1}{x - x + i\epsilon} (\dots H(x, x, t))$$

$$\mathbb{T}^{\text{BH}} \text{ real} \Rightarrow \Delta\sigma \sim \mathbb{T}^{\text{BH}} \int m (\mathbb{T}^{\text{DVCS}})$$

$\Rightarrow \Delta\sigma$ measures $H(x=x, x, t) \dots$

$\rightarrow \Delta\sigma$ useless for the sum rules $\int dx H(x, x, t)$



(Unless x dependence is under control...)

Use of Cebsaf is limited to:

- * Test scaling along $x = x$ (no real part)
- * Test models along $x = x$

• DVCS potentially powerful probe of nuclear structure

• Points to clarify:

- how $R(\Gamma_{VSS})$ approaches scaling?
- how to extrapolate some rules to $t=0$
- is the experiment feasible?

